# Are Students Practicing What We Think They Are? Refining Knowledge Components in Calculus with AI

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**ABSTRACT:** Learning analytics relies on knowledge-component (KC) mappings to identify which skills students practice, yet scaffolded activities may assign the same target skill to steps requiring different operations. We introduce an AI-assisted, step-level KC refinement approach guided by cognitive task analysis principles to better align skill assignments with the demands of individual learning steps. Applied to 534 questions from an online calculus course, the approach revises KC assignments for 417 questions while preserving complete coverage. In a Product Rule activity, only six of 13 questions require Product Rule recognition or application, compared with all 13 under the original mapping. A human-labeled test set shows exact agreement on 22 KC decisions ($\kappa = 1.0$). Using 155,111 observations from 316 learners, DataShop's Additive Factors Model (AFM) Optimized evaluation shows improved model fit (AIC: $-372.86$; BIC: $-234.29$) and predictive accuracy (lower student-blocked and item-blocked RMSE). These findings suggest that aligning KC assignments with the skills students are required to practice, rather than the skills activities are intended to teach, can improve cognitive-model fit and prediction while providing actionable insights for instructional design.

**Keywords:** Knowledge components; cognitive model refinement; AI-assisted learning analytics; calculus; instructional design

## 1 INTRODUCTION

Learning analytics relies on knowledge components (KCs) to identify which skills students practice as they work through learning activities. However, in scaffolded instruction, the skills required at each step may differ from the activity's overall learning goal. For example, a calculus activity designed to teach the Product Rule may first ask students to apply the Power Rule. If every step inherits the Product Rule KC label, prerequisite work may be incorrectly counted as Product Rule practice, affecting the accuracy of skill-specific learning curves and practice histories.

Previous research explored improving knowledge-component (KC) models through statistical methods (Cen et al., 2006), human-machine collaboration (Stamper & Koedinger, 2011), and automated KC identification (Moon et al., 2025). Building on these works, we refine KC assignments to reflect the skills students are required to practice at each step, rather than the skills an activity is intended to teach. We investigate whether this closer alignment between KC labels and task demands also improves the fit and predictive power of cognitive models for student performance. We ask: **Can step-level KC refinement reveal mismatches between instructional goals and required skills and improve cognitive-model fit and predictive accuracy? How can these insights inform future practice design?**

## 2 DATA AND METHODS

This case study uses student log data from an online learning platform deployed in more than 30 schools. We focus on Calculus Chapter 3 (Derivatives), comprising 534 questions across nine modules. The sampled dataset, obtained from DataShop (Koedinger et al., 2010), includes 155,111 student-step observations from 316 learners. First attempts were coded as correct (1) or incorrect (0), with hint requests counted as incorrect. The original KC mapping contains 53 KCs and 981 question–KC pairs.

**Step 1 Factor derivatives**
Given: $f(x) = x^2 + 7x$ $g(x) = 5x - 3$
Find: $f'(x) = ?$ $g'(x) = ?$
**Original KCs:** Power Rule, Product Rule, Quotient Rule, Sum Rule
**Refinement:** Keep Power Rule + Sum Rule. Remove Product Rule + Quotient Rule.

**Step 2 Product derivative**
Given: $f(x) = x^2 + 7x$ $g(x) = 5x - 3$
$f'(x) = 2x + 7$ $g'(x) = 5$
Find: $(fg)' = ?$
Correct answer: $(fg)' = f'g + fg' = (2x + 7)(5x - 3) + (x^2 + 7x)(5)$
**Original KCs:** Power Rule, Product Rule, Sum Rule
**Refinement:** Keep Product Rule. Remove Power Rule + Sum Rule.

**Figure 1. Two successive steps within a Product Rule activity. Step 1 asks learners to differentiate the two factors. Step 2 supplies the derivatives and asks learners to find the derivative of their product. KC assignments distinguish prerequisite differentiation from Product Rule application.**

**KC refinement guideline development.** We developed guidelines grounded in rational cognitive task analysis to identify the skills required at each step, distinguishing provided information from operations students must perform. For example, Figure 1 shows how prerequisite differentiation steps retain Power Rule and Sum Rule KCs, while Product Rule application retains the Product Rule KC. We allow multiple KCs when a step requires multiple skills and use prompts and hints to identify the intended solution approach rather than labeling every possible approach.

**AI-assisted refinement.** We then used GPT-6.1-sol Codex agents to apply these guidelines across all 534 questions. The agents examined question prompts, hints, answer choices, and instructional context, then retained, removed, or added KC assignments using the full KC catalog.

**Human-labeled test set validation.** To evaluate the AI-assisted refinement, two authors independently labeled 22 step–KC decisions across six response steps from two multi-step Product Rule problems without seeing AI or each other's labels. The agents' predictions matched both authors' labels exactly (100% agreement; all pairwise Cohen's $\kappa = 1.0$).

**KC model evaluation.** We evaluated both KC mappings using DataShop's Additive Factors Model (AFM) Optimized on 155,111 student-step observations. We assessed model fit using Akaike information criterion (AIC) and Bayesian information criterion (BIC), which balance model fit against complexity (Cen et al., 2006), and predictive accuracy using cross-validation root mean squared error (RMSE) (Stamper & Koedinger, 2011). Lower values indicate better performance for these metrics.

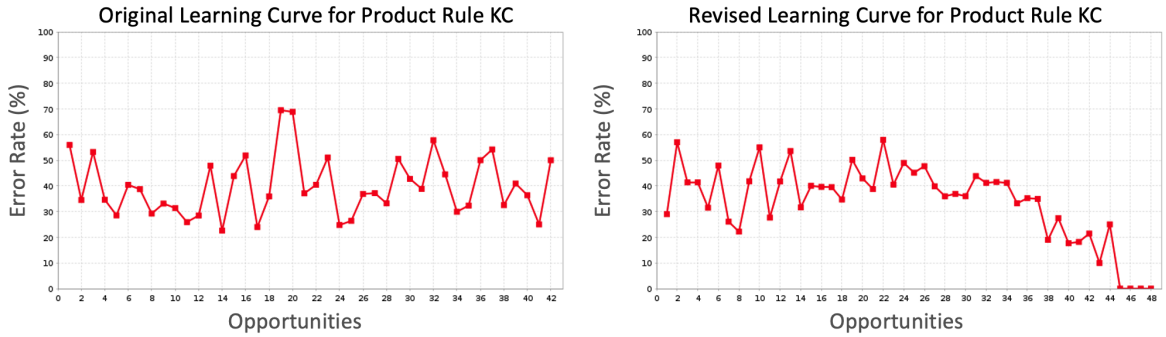



**Figure 2. DataShop learning curves for the Product Rule KC before (left) and after (right) step-level refinement. By aligning KC labels with the skills required at each step, the revised learning curve better reflects students' Product Rule practice, revealing a clearer decline in error rates across practice opportunities.**

## 3 RESULTS

The revised mapping changes KC assignments for 417 of the 534 questions while maintaining complete question coverage. In the Product Rule example (Figure 1), only six of the 13 steps require Product Rule recognition or application, compared with all 13 under the original mapping. This refinement changes which responses count as Product Rule practice, revealing a clearer decline in error rates across successive practice opportunities in the revised learning curve (Figure 2). Across the chapter, the revised mapping contains 60 KCs and 984 question–KC pairs, compared with 53 KCs and 981 pairs originally. The revised KC mapping also achieves better model fit, as indicated by lower AIC and BIC, and higher predictive accuracy, as indicated by RMSE (Table 1).

**Table 1. Original vs. revised KC mappings: AFM Optimized results (155,111 observations).**

| Measure | Original | Revised | Difference |
|---|---|---|---|
| AIC | 176,063.34 | 175,690.48 | $-372.86$ |
| BIC | 180,240.41 | 180,006.12 | $-234.29$ |
| Student-blocked RMSE | 0.483349 | 0.482950 | $-0.000399$ |
| Item-blocked RMSE | 0.453854 | 0.453450 | $-0.000404$ |

## 4 DISCUSSION AND CONCLUSION

Our findings suggest that cognitive models better fit and predict student responses when KC assignments reflect the skills required by each step rather than the activity's intended learning goals. By distinguishing prerequisite practice from target-skill practice, the revised mapping also reveals where activities may provide less practice of the intended skill than their original labels suggest. For example, in Figure 1, students first compute $f'(x)$ and $g'(x)$ using prerequisite Power Rule skills before applying the Product Rule. If the instructional goal is to strengthen Product Rule mastery, educators could provide these derivatives directly and replace the prerequisite steps with additional Product Rule exercises. Alternatively, prerequisite practice could be retained for students who have not yet mastered the Power Rule. Future studies could evaluate whether such changes improve instructional efficiency and later independent Product Rule performance.

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